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Gluon Radiation Off Scalar Stop Particles

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ABSTRACT

We present the distributions for gluon radiation off stop-antistop particles produced in e^+e^- annihilation: $e^+e^- \to \tilde{t}\,\bar{\tilde{t}}\,g$. For high energies the splitting functions of the fragmentation processes $\tilde{t} \to \tilde{t}\,g$ and $g \to \tilde{t}\,\bar{\tilde{t}}$ are derived; they are universal and apply also to high-energy stop particles produced at hadron colliders.

Introduction. Stop particles are exceptional among the supersymmetric partners of the standard-model fermions. Since the top quarks are heavy, the masses of the two stop particles $\tilde{t}_1$ and $\tilde{t}_2$, mixtures of the left (L) and right (R) squarks, may split into two levels separated by a large gap [1]-[3]. The mass of the lightest eigenstate $\tilde{t}_1$ could be so low that the particle may eventually be accessible at the existing $p\bar{p}$ and even e^+e^- storage rings. So far the result of search experiments at e^+e^- colliders [4, 5] has been negative and a lower limit of 45.1 GeV has been set at LEP [5] for the L/R mixing angle outside the band of $\cos^2\theta_t$ between 0.17 and 0.44 and for a mass difference between the $\tilde{t}_1$ and the lightest neutralino $\tilde{\chi}_1^0$ of more than 5 GeV. The higher energy at LEP2 and dedicated efforts at the Tevatron will open the mass range beyond the current limits soon.

To begin, we briefly summarize the well-known theoretical predictions for the cross section of the production process [Fig.1(a)]

$$e^+ \, e^- \to \tilde{t}_1 \, \bar{\tilde{t}}_1$$

For a given value θ_t of the L/R mixing angle, the vertices of the $\tilde{t}_1$ pair with the photon and the Z boson may be written as $ie_0\tilde{Q}[p_{\tilde{t}_1} - p_{\bar{\tilde{t}}_1}]_\mu$, where $p_{\tilde{t}_1}$ and $p_{\bar{\tilde{t}}_1}$ are the 4-momenta of the stop and antistop squarks, and the charges read

$$\begin{aligned} \tilde{Q}_\gamma &= -e_t \\ \tilde{Q}_Z &= (\cos^2\theta_t - 2e_t\sin^2\theta_W)/\sin 2\theta_W \end{aligned}$$

respectively. θ_W is the standard electroweak mixing angle and $e_0 = \sqrt{4\pi\alpha}$ is the electromagnetic coupling to be evaluated with $\alpha^{-1}(M_Z) = 129.1$ in the improved Born approximation [6]. The Z boson coupling vanishes for the L/R mixing angle $\cos^2\theta_t \to 2e_t\sin^2\theta_W \approx 0.30$. Defining the γ and Z vector/axial-vector charges of the electron, as usual, by $e_e = -1$, $v_e = -1 + 4\sin^2\theta_W$ and $a_e = -1$, the cross section can be expressed in the compact form [2]

$$\begin{aligned} \sigma_B[e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1] &= \frac{\pi\alpha^2}{s}\left[\tilde{Q}_\gamma^2 + \frac{(v_e^2+a_e^2)\tilde{Q}_Z^2}{4\sin^2 2\theta_W}\frac{s^2}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\right. \\ &\qquad\left. + \frac{v_e\tilde{Q}_\gamma\tilde{Q}_Z}{\sin 2\theta_W}\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\right]\beta^3 \end{aligned} \tag{1}$$

where $\sqrt{s}$ is the center of mass energy and M_Z, Γ_Z are the mass and the total width of the Z boson, respectively. The P-wave excitation near the threshold gives rise to the familiar β^3 suppression, where $\beta = (1 - 4m_{\tilde{t}_1}^2/s)^{1/2}$ is the velocity of the stop particles. Angular momentum conservation enforces the $\sin^2\theta$ law, $\sigma_B^{-1}d\sigma_B/d\cos\theta = \frac{3}{4}\sin^2\theta$, for the angular distribution of the stop particles with respect to the beam axis.

QCD corrections. Gluonic corrections modify the cross section [7, 8][1]. The virtual corrections, Fig.1(b), can be expressed by the form factor

[1]Since we focus on QCD gluon effects for light stop particles in the LEP range, we do not take into account quark-gluino loop effects, assuming the gluino to be heavy; these loop effects have been discussed for squark production at the Tevatron in Ref.[9] and at e^+e^- colliders in Ref.[10].

$$
\begin{aligned}
F(s) &= \frac{4}{3}\frac{\alpha_s}{\pi}\left\{\frac{s-2m_{\tilde{t}_1}^2}{s\beta}\left[2\,\mathrm{Li}_2(w)+2\log(w)\log(1-w)-\frac{1}{2}\log^2(w)+\frac{2}{3}\pi^2-2\log(w)\right.\right.\\
&\qquad\left.\left.-\log(w)\log\left(\frac{\lambda^2}{m_{\tilde{t}_1}^2}\right)\right]-2-\log\left(\frac{\lambda^2}{m_{\tilde{t}_1}^2}\right)\right\} \qquad (2)
\end{aligned}
$$

where α_s is the strong coupling constant and the kinematical variable w is defined as $w = (1-\beta)/(1+\beta)$. The form factor is infrared (IR) divergent. We have regularized this divergence by introducing a small parameter λ for the gluon mass. The IR singularity is eliminated by adding the contribution of the soft gluon radiation [Fig.1(c)], with the scaled gluon energy integrated up to a cut-off value $\epsilon_g = 2E_g^{cut}/\sqrt{s} \ll 1$. The sum of the virtual correction (V) and the soft-gluon radiation (S) depends only on the physical energy cut-off ϵ_g,

$$
\begin{aligned}
\sigma_{V+S} &= \sigma_B\frac{4}{3}\frac{\alpha_s}{\pi}\left\{\frac{s-2m_{\tilde{t}_1}^2}{s\beta}\left[4\,\mathrm{Li}_2(w)-2\log(w)\log(1+w)+4\log(w)\log(1-w)\right.\right.\\
&\qquad\left.\left.+\frac{1}{3}\pi^2-2\log(w)\log(\epsilon_g)\right]+\frac{4m_{\tilde{t}_1}^2-3s}{s\beta}\log(w)+\log\left(\frac{m_{\tilde{t}_1}^2}{s}\right)-2\log(\epsilon_g)-2\right\}
\end{aligned}
$$

After including the hard gluon radiation, the dependence on the cut-off ϵ_g disappears from the total cross section. The total QCD corrections can finally be summarized in a universal factor [8]

$$
\sigma[e^+e^- \to \tilde{t}_1\,\bar{\tilde{t}}_1\,(g)] = \sigma_B\left[1+\frac{4}{3}\frac{\alpha_s}{\pi}f(\beta)\right] \qquad (3)
$$

with (Fig.2)

$$
\begin{aligned}
f(\beta) &= \frac{1+\beta^2}{\beta}\Big[4\,\mathrm{Li}_2(w)+2\,\mathrm{Li}_2(-w)+2\log(w)\log(1-w)+\log(w)\log(1+w)\Big]\\
&\quad -4\log(1-w)-2\log(1+w)+\left[3+\frac{1}{\beta^3}\left(2-\frac{5}{4}(1+\beta^2)^2\right)\right]\log(w)+\frac{3}{2}\frac{1+\beta^2}{\beta^2}
\end{aligned}
$$

Very close to the threshold the Coulombic gluon exchange between the slowly moving stop particles generates the universal Sommerfeld rescattering singularity [11] $f \to \pi^2/2\beta$, which damps the threshold suppression, yet does not neutralize it entirely. Employing methods based on non-relativistic Green's functions, an adequate description of stop pair production near threshold has been given in Ref.[12], which also takes into account screening effects due to the finite decay width of the stop particles. In the high-energy limit [8] the correction factor in eq.(3) approaches the value $(1+4\alpha_s/\pi)$.

In this note we present a general analysis of hard gluon radiation. We also include stop fragmentation due to collinear gluon emission in the perturbative regime at high energies and we give an account of non-perturbative fragmentation effects.

For unpolarized lepton beams the cross section for gluon radiation off $\tilde{t}_1$ squarks

$$e^+ e^- \to \tilde{t}_1 \bar{\tilde{t}}_1 g$$

depends on four variables: the polar angle θ between the momentum of the $\tilde{t}_1$ squark and the e^- momentum, the azimuthal angle χ between the $\tilde{t}_1 \bar{\tilde{t}}_1 g$ plane and the plane spanned by the $e^\pm$ beam axis with the $\tilde{t}_1$ momentum [see Ref.[13]], and two of the scaled energies $x(\tilde{t}_1)$, $\bar{x}(\bar{\tilde{t}}_1)$, $z(g)$ in units of the beam energy. The energies are related through $x + \bar{x} + z = 2$ and vary over the intervals $\mu \leq x, \bar{x} \leq 1$ and $0 \leq z \leq 1 - \mu^2$, where $\mu = 2m_{\tilde{t}_1}/\sqrt{s}$ denotes the squark mass in units of the beam energy. For the angles between the squark and gluon momenta we have

$$\begin{aligned}
\cos\theta_{\tilde{t}_1 \bar{\tilde{t}}_1} &= \frac{2 - 2(x+\bar{x}) + x\bar{x} + \mu^2}{\sqrt{(x^2-\mu^2)(\bar{x}^2-\mu^2)}} \\
\cos\theta_{\tilde{t}_1 g} &= \frac{2 - 2(x+z) + xz}{z\sqrt{x^2-\mu^2}}
\end{aligned}$$

The spin-1 helicity analysis of the cross section results in the following well-known angular decomposition [14]

$$\begin{aligned}
\frac{d\sigma}{dx\, d\bar{x}\, d\cos\theta\, d\chi/2\pi} &= \frac{3}{8}(1+\cos^2\theta)\frac{d\sigma^U}{dx\, d\bar{x}} + \frac{3}{4}\sin^2\theta \frac{d\sigma^L}{dx\, d\bar{x}} \\
&\quad - \frac{3}{2\sqrt{2}}\sin 2\theta \cos\chi \frac{d\sigma^I}{dx\, d\bar{x}} + \frac{3}{4}\sin^2\theta\cos 2\chi \frac{d\sigma^T}{dx\, d\bar{x}}
\end{aligned} \tag{4}$$

[U = transverse (no flip), L = longitudinal, I = trv∗long, T = trv∗trv (flip)]. If the polar and azimuthal angles are integrated out, the cross section is given by $\sigma = \sigma^U + \sigma^L$.

It is convenient to write the helicity cross sections as

$$\frac{\beta^3}{\sigma_B}\frac{d\sigma^j}{dx\, d\bar{x}} = \frac{\alpha_s}{4\pi}\frac{S^j + \mu^2 N^j}{(1-x)(1-\bar{x})} \tag{5}$$

The densities S^j and N^j are summarized in Table 1; p is the momentum of the $\tilde{t}_1$ squark, $\bar{p}$ and k are the longitudinal momenta of $\bar{\tilde{t}}_1$ and g in the $\tilde{t}_1$ direction, and p_T is the modulus of the transverse $\bar{\tilde{t}}_1$, g momentum with respect to this axis [all momenta in units of the beam energy]. Since I, T correspond to γ, Z helicity flips by 1 and 2 units, they are of order p_T and p_T^2, respectively. Note that the threshold suppression is absent in the U, I, T components and attenuated in the leading longitudinal L term as expected from eq.(3).

Fragmentation. In the limit where the gluons are emitted from fast moving squarks with small angles, the gluon radiation

$$\tilde{t}_1 \to \tilde{t}_1 g$$

can be interpreted as a perturbative fragmentation process. From the form of the differential cross section $d\sigma/dz\, dp_T^2$ we find in this limit for the splitting functions, in analogy to the

	S^j	N^j
U	$\frac{32}{3}\,(1-x)\,(1-\bar{x})$	$-\frac{4}{3}\,p_T^2\,\frac{1-x}{1-\bar{x}}$
L	$\frac{16\beta^2}{3}\,(1-z)$	$\frac{4}{3}\,\left[p_T^2\,\frac{1-x}{1-\bar{x}}-\beta^2\left(\frac{1-x}{1-\bar{x}}+\frac{1-\bar{x}}{1-x}+2\right)\right]$
I	$-\frac{4\sqrt{2}}{3}\,p_T\,p$	$\frac{2\sqrt{2}}{3}\,p_T\left(p-\bar{p}\,\frac{1-x}{1-\bar{x}}\right)$
T	0	$\frac{2}{3}\,p_T^2\,\frac{1-x}{1-\bar{x}}$

Table 1: Coefficients of the helicity cross sections in eq.(5). The energy and momentum variables are defined in the text.

Weizsäcker-Williams [15] and Altarelli-Parisi splitting functions [16],

$$\begin{aligned} P[\tilde{t}_1 \to \tilde{t}_1; x] &= \frac{\alpha_s}{2\pi}\,\frac{8}{3}\,\frac{x}{1-x}\,\log\frac{Q^2}{m^2_{\tilde{t}_1}} \\ P[\tilde{t}_1 \to g; z] &= \frac{\alpha_s}{2\pi}\,\frac{8}{3}\,\frac{1-z}{z}\,\log\frac{Q^2}{m^2_{\tilde{t}_1}} \end{aligned} \tag{6}$$

As usual, x and z are the fractions of energy transferred from the $\tilde{t}_1$ beam to the squark $\tilde{t}_1$ and the gluon g after fragmentation, respectively; Q is the evolution scale of the elementary process, normalized by the squark mass rather than the QCD Λ parameter [in contrast to the light quark/gluon sector]. As a consequence of angular-momentum conservation, the gluon cannot pick up the total momentum of the squark beam. [Similar zeros have been found for helicity-flip fragmentation functions in QED/ QCD [16, 17].]

By using the crossing rules $\{z \to 1, 1 \to x\}$ and $\{1-x \leftrightarrow 1-x\}$, familiar from the analogous splitting functions in QED [18], we derive for the elementary gluon splitting process into a squark-antisquark pair

$$g \to \tilde{t}_1\bar{\tilde{t}}_1$$

the distribution

$$P[g \to \tilde{t}_1; x] = \frac{\alpha_s}{2\pi}\,\frac{1}{2}\,x\,(1-x)\,\log\frac{Q^2}{m^2_{\tilde{t}_1}} \tag{7}$$

after adjusting color and spin coefficients properly. This splitting function is symmetric under the $\tilde{t}_1 \leftrightarrow \bar{\tilde{t}}_1$ exchange, i.e. $\{x \leftrightarrow 1-x\}$. The probability is maximal for the splitting into equal fractions $x = 1/2$ of the momenta, in contrast to spinor QED/QCD where the splitting into a quark-antiquark pair is proportional to $x^2 + (1-x)^2$ and hence asymmetric configurations are preferred.

The above splitting functions provide the kernels for the shower expansions in perturbative QCD Monte Carlos for e^+e^- annihilation such as Pythia [19] and Herwig [20]. They serve

the same purpose in the hadron-hadron versions of these generators as well as Isajet [21]. Of course, the interpretation of the radiation processes as universal fragmentation processes becomes increasingly adequate with rising energy of the fragmenting squarks/gluons.

If the $\tilde{t}_1$ squark is lighter than the top quark, the lifetime will be long, $\tau \geq 10^{-20}$sec, since the dominant decay channel $\tilde{t}_1 \to t + \tilde{\chi}_1^0$ is shut off [$\tilde{\chi}_1^0 = LSP$]. The decay widths corresponding to the 2-body decay $\tilde{t}_1 \to c + \tilde{\chi}_1^0$ and 3-body slepton decays involve the electroweak coupling twice and hence will be very small [2]. As a result, the lifetime is much longer than the typical non-perturbative fragmentation time of order 1 fm [i.e. $\mathcal{O}\,(10^{-23}$sec)] so that the squark has got enough time to form $(\tilde{t}_1\bar{q})$ and $(\tilde{t}_1 qq)$ fermionic and bosonic hadrons. However, the energy transfer due to the non-perturbative fragmentation, evolving after the early perturbative fragmentation, is very small as a result of Galilei's law of inertia. Describing this last step in the hadronization process of a $\tilde{t}_1$ jet by the non-perturbative fragmentation function *à la* Peterson et al. [22] (which accounts very well for the heavy-quark analogue), we find

$$D(x)^{NP} \approx \frac{4\sqrt{\epsilon}}{\pi} \frac{1}{x\,[1 - 1/x - \epsilon/(1-x)]^2} \tag{8}$$

with the parameter $\epsilon \sim 0.5\,\mathrm{GeV}^2/m_{\tilde{t}_1}^2$. Here, $x = E[(\tilde{t}_1\,\bar{q})]/E[\tilde{t}_1]$ is the energy fraction transferred from the $\tilde{t}_1$ parton to the $(\tilde{t}_1\,\bar{q})$ hadron etc. The resulting average non-perturbative energy loss

$$< 1 - x >^{NP} \sim \frac{2\sqrt{\epsilon}}{\pi}\left[\log\left(\frac{1}{\epsilon}\right) - 3\right]$$

is numerically at the level of a few percent.

Monte Carlo programs for the hadronization of $\tilde{t}_1$ squarks link the early perturbative fragmentation with the subsequent non-perturbative hadronization. The relative weight of perturbative and non-perturbative fragmentation can be characterized by the average energy loss in the two consecutive steps. The overall retained average energy of the $\tilde{t}_1$ squarks factorizes into the two components,

$$< x > = < x >^{NP} < x >^{PT} \tag{9}$$

Summing up the energy loss due to multiple gluon radiation at high energies, we find in analogy to heavy-quark fragmentation [23]

$$< x >^{PT} = \left[\frac{\alpha_s(m_{\tilde{t}_1}^2)}{\alpha_s(E^2)}\right]^{-8/3b}$$

with $b = (11 - 2n_f/3) + (-2 - n_f/3)$ being the LO QCD β function including the colored supersymmetric particle spectrum. At high energies, the perturbative multi-gluon radiation has a bigger impact than the final non-perturbative hadronization mechanism, e.g. $< x >^{PT} \approx 0.93$ for a $\tilde{t}_1$ beam energy $E = 1\,\mathrm{TeV}$ and $m_{\tilde{t}_1} = 200\,\mathrm{GeV}$ as compared to $< x >^{NP} \approx 0.98$. At low energies the two fragmentation effects are of comparable size.

After finalizing the manuscript, we received a copy of Ref.[10] in which the total cross sections for squark pair production in e^+e^- annihilation have been discussed including squark-gluon and quark-gluino loops, yet not the gluon-jet distributions analysed in the present note.

We thank our colleagues at the LEP2 Workshop who demanded the analysis presented here to refine the experimental stop search techniques.

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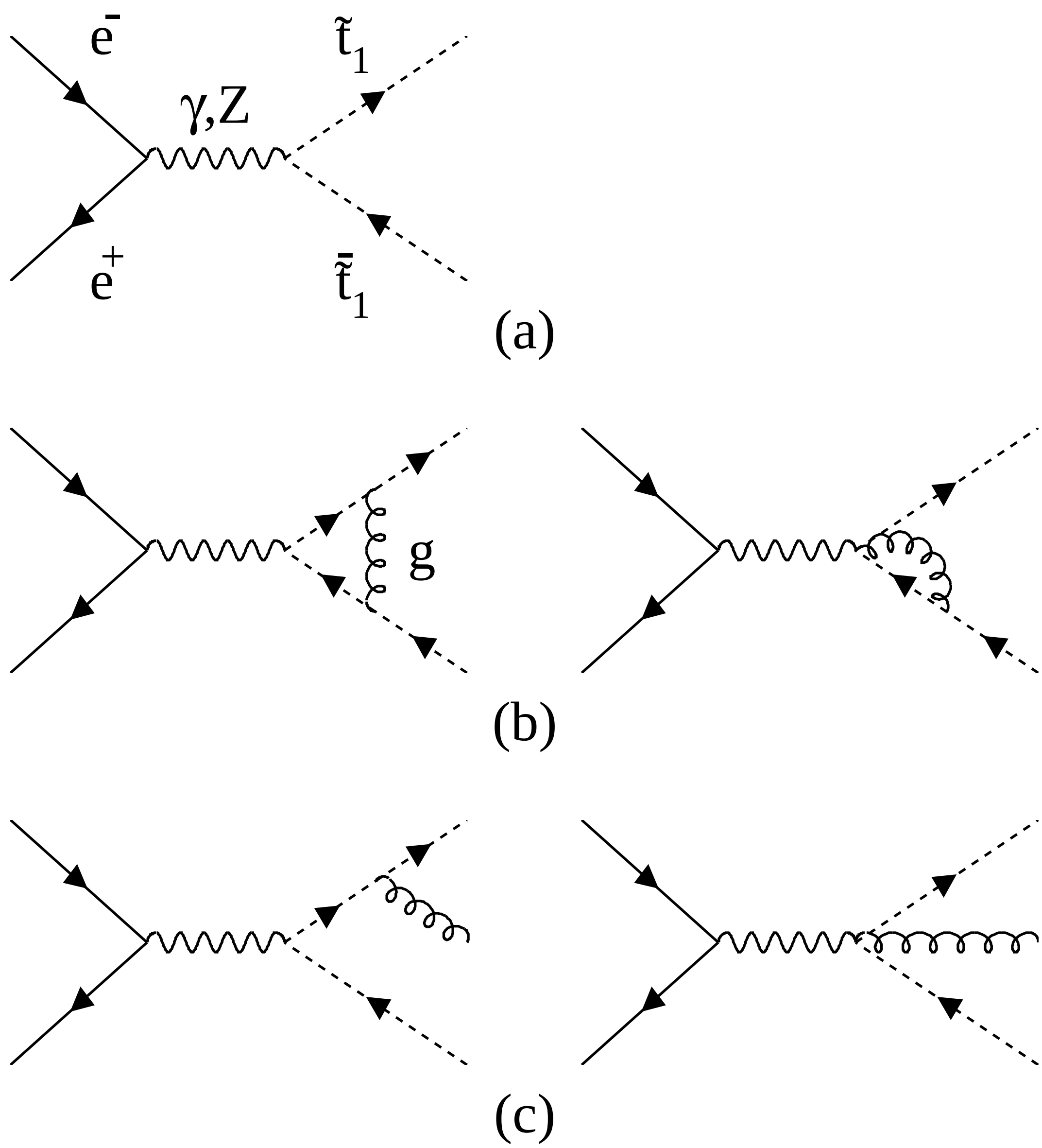

Figure 1: Generic diagrams for $\tilde{t}_1\bar{\tilde{t}}_1$ production in e^+e^- collisions. (a) Born level; (b) virtual QCD corrections; (c) gluon radiation.

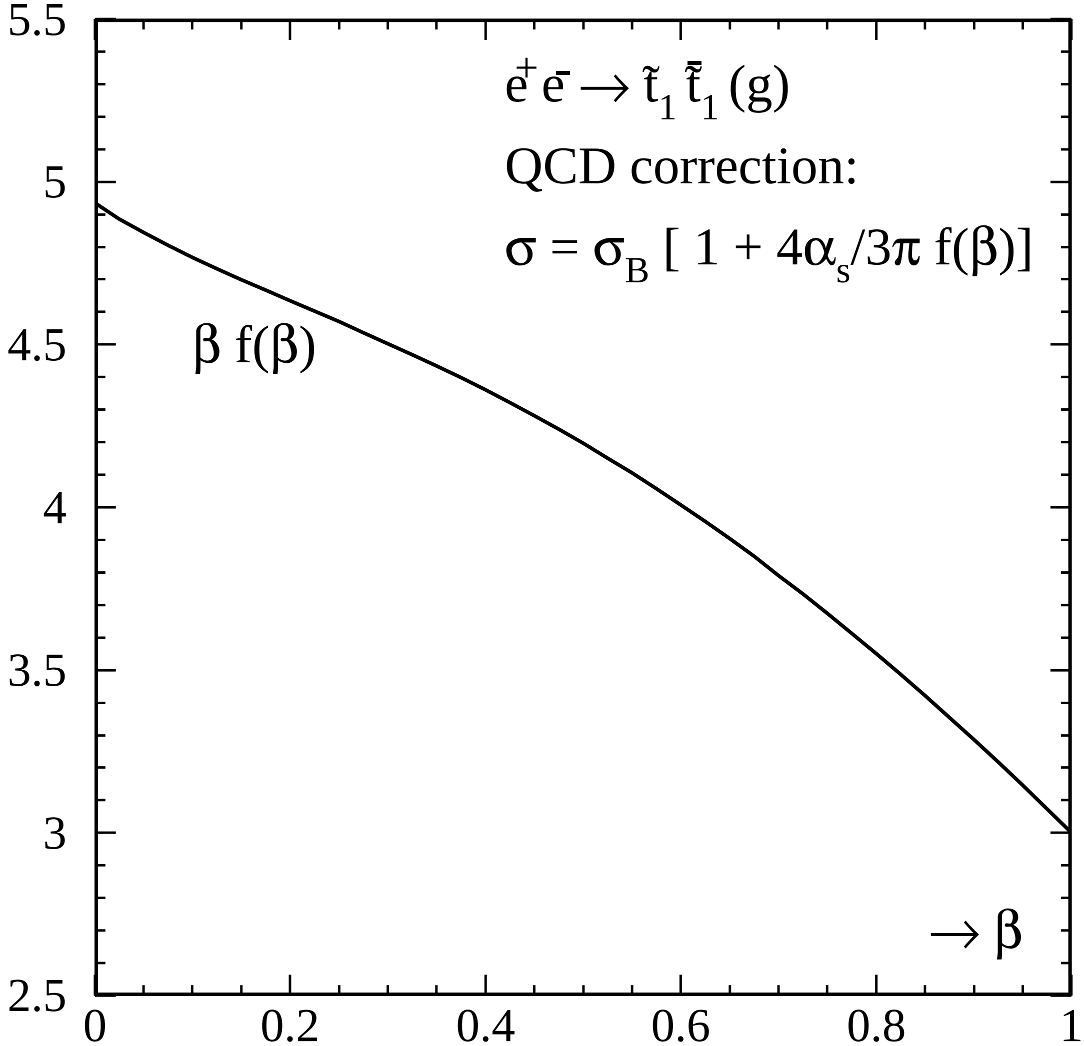

Figure 2: Coefficient of the QCD correction to the total cross section; shown is $\beta f(\beta)$, cf. eq.(3), with $\beta = (1 - 4m_{\tilde{t}_1}^2/s)^{1/2}$.